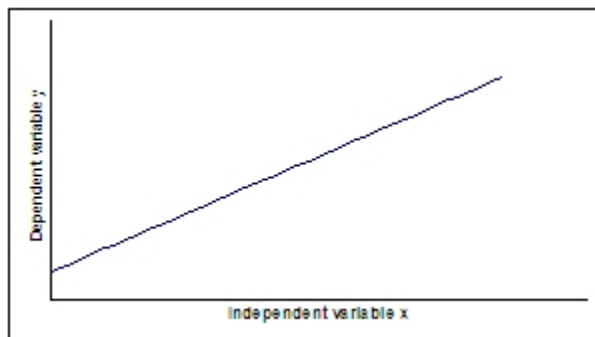


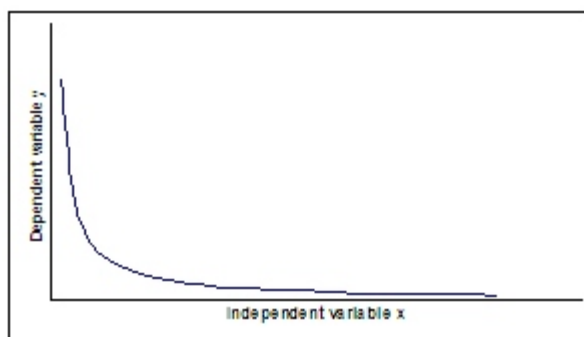
Proportional Relationships

Four common patterns that can be seen when graphing data are:

**A** Linear

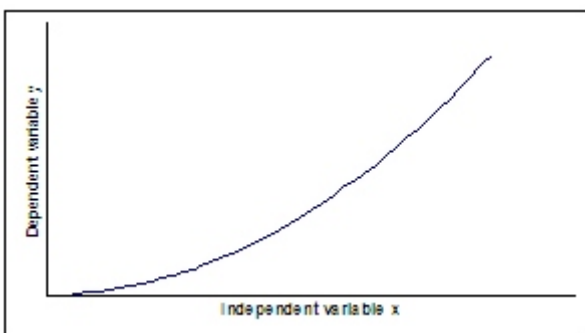
$$y \propto x$$

$$y = kx$$

**B** Inverse

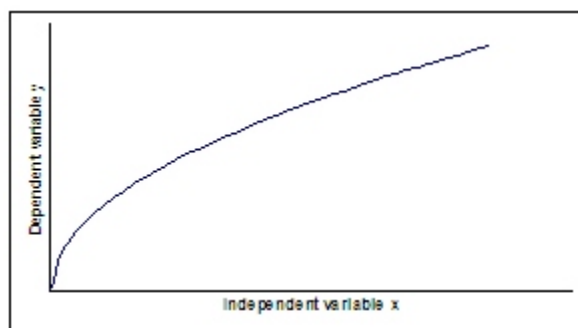
$$y = \frac{1}{x^n} \text{ or } y \propto x^{-n}$$

$$y = kx^{-n}$$

**C** Power

$$y \propto x^n$$

$$y = kx^n$$

**D** Root

$$y = \sqrt[n]{x} \text{ or } y \propto x^{1/n}$$

$$y = kx^{1/n}$$

Determining the Equation by Linearizing Graphs

You can often produce a straight-line graph from a non-linear relationship by making an appropriate choice of independent variables for the graph. This procedure is called curve straightening and produces an equation of the form

$$\text{quantity on the vertical axis} = m (\text{quantity on the horizontal axis}) + b$$

You can straighten a curve by selecting the quantity graphed on the horizontal axis to match the general type of variation shown by the data. E.g., if the independent variable is x and you suspect

- inverse variation: plot $\frac{1}{x}$, $\frac{1}{x^2}$, or $\frac{1}{x^3}$ on the horizontal axis

- power variation: plot x^2 or x^3 on the horizontal axis
- root variation: plot $x^{1/2}$ or $x^{1/3}$ on the horizontal axis

Procedure

1. Plot the raw data and identify the general relationship shown by the graph.
2. Modify the independent variable to suit the proposed type of relationship. Add the new quantity to your data table and then draw a new graph of y against the quantity derived from x .
3. If the new graph is a straight line, calculate its slope and y -intercept. Use these values to write and simplify an equation to represent the data.
4. If the new graph is not a straight line, repeat steps 2 and 3 using a different modification of the independent variable until you get a straight-line graph.

Example (in class)

A force of 1.96 N was used to accelerate a lab cart with mass 0.225 kg. The mass of the cart was systematically increased, producing the accelerations shown below. Find an equation that represents this data.

Mass (kg)	Acceleration (m/s^2)
0.225	8.71
0.325	6.05
0.425	4.70
0.525	3.73
0.625	3.09
0.725	2.81

Points to Remember

- Make sure the scales on your graph start at 0 or you might see a magnified view of only a small portion of the graph. A graph that is a gentle curve might appear straight.
- Sometimes it is good to look only at a certain part of the data set. Exclude obvious errors that are far away from the best-fit line.
- If you suspect that a certain phenomenon follows an inverse square law, for example, it would be sensible to choose $1/(\text{dependent variable})^2$ as the quantity to graph; then you can determine how closely your observations approach the ideal model.

Problems